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THE EQUATION OF DISPERSION AND THE DISPLACEMENT VECTOR IN THE ANTISYMMETRIC CASE

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Abstract: In this paper we study the propagation of the Lamb waves and we find the equation of dispersion and the equations of the displacement vector in the antisymmetric case.

Keywords: nonlinear eigenvalue problems, second order hyperbolic equation, wave equation.

MSC2010: 35P30, 35L10, 35L05.

1. INTRODUCTION

We are under the same conditions of the article [1], so we consider an elastic, isotropic, continuous and homogeneous medium and we study the propagation of the antisymmetric Lamb waves through it.

Using the results from [1], [2] and [3], we obtain the equation of dispersion and the equations of the displacement vector in the symmetric case.

2. PROBLEM FORMULATION

We consider the normal guided Lamb waves that appear in a plate of thickness $2h$ comparable with the wavelength, due to coupling between the components longitudinal L and the transverse components of the wave TV . Thus, two types of wave Lamb can be produced, but, in this paper, we study the antisymmetric waves which are depicted in Figure 1, where for each side of the middle of the plate, the transverse components are equal and the longitudinal components are opposite.

Just like in the article [4], we assume homogeneous and isotropic elastic plate bounded by two parallel planes located at a short distance $2h$, and we find the equation of dispersion and the equations of the displacement vector, but in the antisymmetric case.

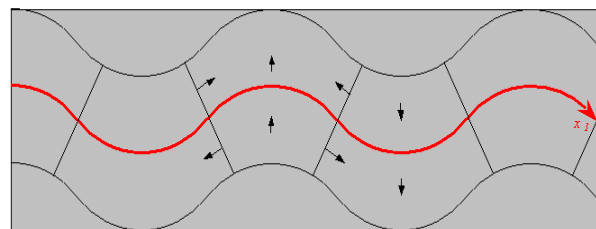


Figure 1: The antisymmetric Lamb waves

3. PROBLEM SOLUTION

In the article [1] we started from the equation of the displacement vector \mathbf{u} for a material point

$$\mathbf{u} = \nabla \varphi + \nabla \times \psi \quad (1)$$

where ϕ is a scalar potential and ψ is a vectorial potential. In expression (1) the two potentials should verify the following two equations of wave

$$\nabla^2 \phi - \frac{1}{v_L^2} \frac{\partial^2 \phi}{\partial t^2} = 0, \quad (2)$$

where

$$v_L = \left(\frac{c_{11}}{\rho} \right)^{\frac{1}{2}} = \left(\frac{\lambda + 2\mu}{\rho} \right)^{\frac{1}{2}}$$

is the phase velocity of the longitudinal waves and

$$\nabla^2 \psi - \frac{1}{v_T^2} \frac{\partial^2 \psi}{\partial t^2} = 0, \quad (3)$$

where

$$v_T = \left(\frac{c_{44}}{\rho} \right)^{\frac{1}{2}} = \left(\frac{\lambda}{\rho} \right)^{\frac{1}{2}}$$

is the phase velocity of the transversal waves.

The scalar and the vectorial potentials are trigonometric functions of time t , with the same frequency ω . However, they can be expressed as follows, with the wave number k :

$$\begin{aligned} \phi &= \phi_0(x_2) e^{i(\omega t - kx_1)}, \\ \psi &= \psi_{0j}(x_2) e^{i(\omega t - kx_1)}, \quad j = 1, 2, 3. \end{aligned} \quad (4)$$

The equation of dispersion has the following relation [1]:

$$\frac{\tan(qh + \alpha)}{\tan(ph + \alpha)} = -\frac{4k^2 pq}{(q^2 - k^2)^2} \quad (5)$$

where the constants p and q are defined as follows:

$$p^2 = \frac{\omega^2}{v_L^2} - k^2, \quad (6)$$

$$q^2 = \frac{\omega^2}{v_T^2} - k^2,$$

and the constant angle $\alpha = 0$ (the symmetric case studied in [4]) and $\alpha = \frac{\pi}{2}$, this is the case that we study in this paper, the antisymmetric case.

Further, using the results from [3], we know that we have three regions with respect to the phase velocity. So, we write the equation of dispersion (5) and the equations of the displacement vector for each subdomain of wave number k taking into account the type of values that can be taken the constants p and q , for the case $\alpha = \frac{\pi}{2}$ (the antisymmetric case).

The first case is obtained for $k < \frac{\omega}{v_L} < \frac{\omega}{v_T} \Leftrightarrow V > v_L > v_T$, and we have p and q real numbers, so the dispersion equation is

$$\frac{\tan\left(qh + \frac{\pi}{2}\right)}{\tan\left(ph + \frac{\pi}{2}\right)} = \frac{\tan(ph)}{\tan(qh)} = -\frac{4k^2 pq}{(q^2 - k^2)^2}, \quad (7)$$

and the equations of the displacement vector are:

$$\begin{aligned} u_1 &= qA \left[-\sin(qx_2) + \frac{2k^2}{k^2 - q^2} \cdot \frac{\sin(qh)}{\sin(ph)} \cdot \sin(px_2) \right] \\ &\quad \cdot \cos(\omega t - kx_1), \\ u_2 &= -kA \left[\cos(qx_2) + \frac{2pq}{k^2 - q^2} \frac{\sin(qh)}{\sin(ph)} \cdot \cos(px_2) \right] \\ &\quad \cdot \sin(\omega t - kx_1). \end{aligned} \quad (8)$$

The second case is obtained for $\frac{\omega}{v_L} < k < \frac{\omega}{v_T} \Leftrightarrow v_L > V > v_T$ and we have p an imaginary number, i.e. $p = ip$ and q a real



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number, so the dispersion equation has the form:

$$\frac{\tan\left(qh + \frac{\pi}{2}\right)}{\tan\left(ph + \frac{\pi}{2}\right)} = \frac{\tan(ph)}{\tan(qh)} = \frac{4k^2 pq}{(q^2 - k^2)^2}, \quad (9)$$

and the equations of the displacement vector are:

$$u_1 = qA \left[-\sin(qx_2) + \frac{2k^2}{k^2 - q^2} \frac{\sin(qh)}{\sinh(ph)} \sinh(px_2) \right] \cdot \cos(\omega t - kx_1),$$

$$u_2 = -kA \left[\cos(qx_2) + \frac{2pq}{k^2 - q^2} \frac{\sin(qh)}{\sinh(ph)} \cosh(px_2) \right] \cdot \sin(\omega t - kx_1). \quad (10)$$

The third case is obtained for $\frac{\omega}{v_L} < \frac{\omega}{v_T} < k \Leftrightarrow v_L > v_T > V$, and we have p and q imaginary numbers, i.e. $p = ip$ and $q = iq$, so the dispersion equation is

$$\frac{\tan\left(qh + \frac{\pi}{2}\right)}{\tan\left(ph + \frac{\pi}{2}\right)} = \frac{\tan(ph)}{\tan(qh)} = \frac{4k^2 pq}{(q^2 + k^2)^2} \quad (11)$$

and the equations of the displacement vector are the following form:

$$u_1 = qA \left[\sinh(qx_2) - \frac{2k^2}{k^2 + q^2} \frac{\sinh(qh)}{\sinh(ph)} \cdot \sinh(px_2) \right] \sin(\omega t - kx_1),$$

$$u_2 = -kA \left[\cosh(qx_2) - \frac{2pq}{k^2 + q^2} \frac{\sinh(qh)}{\sinh(ph)} \cdot \cosh(px_2) \right] \sin(\omega t - kx_1), \quad (12)$$

where A is a constant factor.

3. CONCLUSIONS

In this paper we use the antisymmetric Lamb waves in an elastic, isotropic, continuous and homogeneous medium and we find the equation of dispersion and the equations of the displacement vector for every subdomain of the wave number k in the antisymmetric case.

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